# ITT8060: Advanced Programming (in F#)

## Lecture 3: Lists, Functions, Basic Types and Tuples

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based on slides by Michael R. Hansen

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# Overview

- Lists: values and constructors
- Recursions following the structure of lists

The purpose of this lecture is to give you an (as short as possible) introduction to lists, so that you can solve a problem which can illustrate some of F#'s high-level features.

This part is *not* intended as a comprehensive presentation on lists, and we will return to the topic again later.

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# Lists

A list is a finite sequence of elements having the same type:

$[v_1; \ldots; v_n]$ ([] is called the empty list)

```
[2;3;6];;
val it : int list = [2; 3; 6]

["a"; "ab"; "abc"; ""];;
val it : string list = ["a"; "ab"; "abc"; ""]

[sin; cos];;
val it : (float->float) list = [<fun:...>; <fun:...>]

[(1,true); (3,true)];;
val it : (int * bool) list = [(1, true); (3, true)]

[[]; [1]; [1;2]];;
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# List constructors: `[]` and `::`

Lists are generated as follows:

- `[]` is a list (empty list)
- if `x` is an element and `xs` is a list, then `x :: xs` is a list (non-empty list)

`::` associates to the right, i.e., $x_1 :: x_2 :: xs$ means $x_1 :: (x_2 :: xs)$




Graph for $x_1 :: x_2 :: xs$

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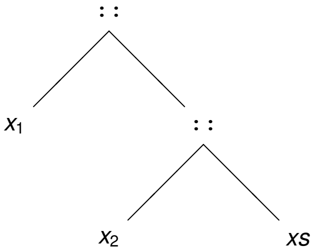


Graph for $x_1 :: x_2 :: xs$

# Trees for lists

A non-empty list $[x_1; x_2; \ldots; x_n]$, $n \geq 1$, consists of

- a *head* $x_1$ and
- a *tail* $[x_2; \ldots; x_n]$




Graph for `[2;3;2]`

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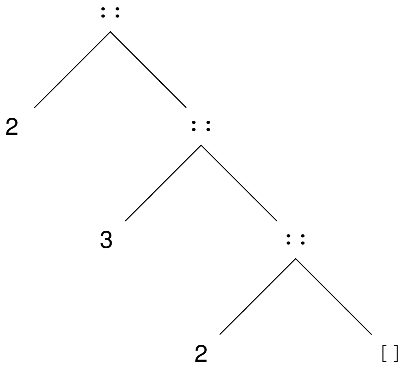


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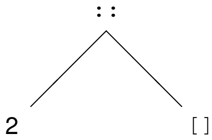


Graph for `[2]`

# Recursion on lists – a simple example

$$\texttt{suml}\ [x_1; x_2; \ldots; x_n] = \sum_{i=1}^{n} x_i = x_1 + x_2 + \cdots + x_n = x_1 + \sum_{i=2}^{n} x_i$$

Constructors are used in list patterns

```
let rec suml = function
  | []    -> 0
  | x::xs -> x + suml xs;;
> val suml : int list -> int
```

```
   suml [1;2]
⇝  1 + suml [2]          (x ↦ 1 and xs ↦ [2])
⇝  1 + (2 + suml [])     (x ↦ 2 and xs ↦ [])
⇝  1 + (2 + 0)           (the pattern [] matches the value [])
⇝  1 + 2
⇝  3
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Recursion follows the structure of lists

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# Outline

- A further look at functions, including higher-order (or curried) functions
- A further look at basic types, including characters, equality and ordering
- A first look at polymorphism
- A further look at tuples and patterns
- A further look at lists and list recursion

Goal: By the end of the day you are acquainted with a major part of the F# language.

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# Anonymous functions

Function expressions with general patterns, e.g.

```
function              // equivalent to ``fun x = match x with’’
 | 2         -> 28   // February
 |4|6|9|11   -> 30   // April, June, September, November
 | _         -> 31   // All other months
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Simple function expressions, e.g.

```
fun r -> System.Math.PI * r * r ;;
val it : float -> float = <fun:clo@10-1>

it 2.0 ;;
val it : float = 12.56637061
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Simple functions expressions with *currying*

$$\texttt{fun}\ x\ y\ \cdots\ z\ \rightarrow e$$

with the same meaning as

$$\texttt{fun}\ x \rightarrow (\texttt{fun}\ y \rightarrow (\cdots\ (\texttt{fun}\ z \rightarrow e)\cdots))$$

For example: The function below takes an integer as argument and returns a function of type `int -> int` as value:

```
fun x y -> x + x*y;;
val it : int ->  int -> int = <fun:clo@2-1>

let f = it 2;;
val f : ( int -> int)

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Functions are first class citizens:
the argument and the value of a function may be functions

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A simple function declaration:

$$\texttt{let}\ f\ x\ =\ e \qquad \text{means} \qquad \texttt{let}\ f\ =\ \texttt{fun}\ x\ \rightarrow e$$

For example: `let circleArea r = System.Math.PI * r * r`

A declaration of a curried function

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Suppose that we have a cube with side length $s$, containing a liquid with density $\rho$. The weight of the liquid is then given by $\rho \cdot s^3$:

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let weight ro s = ro * s ** 3.0;;
val weight : float -> float -> float
```

We can *partially apply* the function to define functions for computing the weight of a cube of either water or methanol:

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let waterWeight = weight 1000.0;;
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# Patterns

We have in previous examples exploited the pattern matching in function expression:

$$\begin{array}{l} \texttt{function} \\ \mid \mathit{pat}_1 \quad \rightarrow \quad e_1 \\ \quad\quad\quad \vdots \\ \mid \mathit{pat}_n \quad \rightarrow \quad e_n \end{array}$$

A match expression has a similar pattern matching feature:

$$\begin{array}{l} \texttt{match}\ e\ \texttt{with} \\ \mid \mathit{pat}_1 \quad \rightarrow \quad e_1 \\ \quad\quad\quad \vdots \\ \mid \mathit{pat}_n \quad \rightarrow \quad e_n \end{array}$$

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# Example

Alternative declarations of the power function:

```
let rec power = function
  | (_,0) -> 1.0
  | (x,n) -> x * power(x,n-1);;
```

are

```
let rec power a = match a with
                  | (_,0) -> 1.0
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and

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let rec power(x,n) = match n with
                     | 0  -> 1.0
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# Infix functions

The prefix version $(\oplus)$ of an infix operator $\oplus$ is a curried function.

For example:

```
(+);;
val it : (int -> int -> int) = <fun:it@1>
```

Arguments can be supplied one by one:

```
let plusThree = (+) 3;;
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plusThree 5;;
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## Function composition: $(f \circ g)(x) = f(g(x))$

For example, if $f(y) = y + 3$ and $g(x) = x^2$, then $(f \circ \ g)(z) = z^2 + 3$.

The infix operator << in F# denotes function composition:

```
let f y = y+3;;          // f(y) = y+3

let g x = x*x;;          // g(x) = x*x

let h = f << g;;         // h = (f o g)
val h : int -> int

h 4;;                    // h(4) = (f o g)(4)
val it : int = 19
```

Using just anonymous functions:

```
((fun y -> y+3) << (fun x -> x*x)) 4;;
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Type of (<<) ?

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# Basic Types: equality and ordering

The basic types: integers, floats, booleans, and strings type were covered last week. Characters are considered on the next slide.
For these types (and many other) equality and ordering are defined.

In particular, there is a function:

$$\texttt{compare}\ x\ y \;=\; \begin{cases} > 0 & \text{if } x > y \\ 0 & \text{if } x = y \\ < 0 & \text{if } x < y \end{cases}$$

For example:

```
compare 7.4 2.0;;
val it : int = 1

compare "abc" "def";;
val it : int = -3

compare 1 4;;
val it : int = -1
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```

# Pattern matching with guards

It is often useful to have when guards in patterns:

```
let ordText x y = match compare x y with
                  | t when t > 0 -> "greater"
                  | 0            -> "equal"
                  | _            -> "less";;

ordText "abc" "Abc";;
val it : string = "greater"
```

The first clause is only taken when `t > 0` evaluates to true.

# Polymorphism and comparison

The type of `ordText`

```
val ordText : 'a -> 'a -> string when 'a : comparison
```

contains

- a type variable `'a`, and
- a type constraint `'a : comparison`

The type variable can be instantiated to any type provided comparison is defined for that type. It is called a polymorphic type.

For example:

```
ordText true false;;
val it : string = "greater"

ordText (1,true) (1,false);;
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... '('a -> 'a)' does not support the 'comparison' ...
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# Characters

Type name: `char`

Values `'a'`, `' '`, `'\''` (escape sequence for `'`)

Examples

```
let isLowerCaseVowel ch =
     System.Char.IsLower ch &&
     (ch='a' || ch='e' || ch = 'i' || ch='o' || ch = 'u');;
val isLowerCaseVowel : char -> bool

isLowerCaseVowel 'i';;
val it : bool = true

isLowerCaseVowel 'I';;
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```

The *i*'th character in a string is achieved using the "dot"-notation:

```
"abc".[0];;
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# Overloaded Operators and Type inference

A squaring function on integers:

| Declaration | Type | |
|---|---|---|
| `let square x = x * x` | `int -> int` | Default |

A squaring function on floats: `square: float -> float`

| Declaration | |
|---|---|
| `let square(x:float) = x * x` | Type the argument |
| `let square x:float = x * x` | Type the result |
| `let square x = x * x: float` | Type expression for the result |
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You can mix these possibilities

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# Tuples

An ordered collection of $n$ values $(v_1, v_2, \ldots, v_n)$ is called an $n$-tuple

Examples

| | |
|---|---|
| `(3, false);`<br>`val it = (3, false) : int * bool` | 2-tuples (pairs) |
| `(1, 2, ("ab",true));`<br>`val it = (1, 2, ("ab", true)) :` ? | 3-tuples (triples) |

Equality defined componentwise, ordering lexicographically

```
(1, 2.0, true) = (2-1, 2.0*1.0, 1<2);;
val it = true : bool
```

```
compare (1, 2.0, true) (2-1, 3.0, false);;
val it : int = -1
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# Tuple patterns

**Extract components of tuples**

```
let ((x,_),(_,y,_)) = ((1,true),("a","b",false));;
val x : int = 1
val y : string = "b"
```

**Pattern matching yields bindings**

**Restriction**

```
let (x,x) = (1,1);;
...
 ...  ERROR ...  'x' is bound twice in this pattern
```

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let ((x,_),(_,y,_)) = ((1,true),("a","b",false));;
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```

## Pattern matching yields bindings

## Restriction

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let (x,x) = (1,1);;
...
 ... ERROR ... 'x' is bound twice in this pattern
```

# Local declarations

Examples

```
let g x =
   let a = 6
   let f y = y + a
   x + f x;;
val g : int -> int

g 1;;
val it : int = 8
```

Note: a and f are not visible outside of g

# Declaration of types and exceptions

Example: Solve $ax^2 + bx + c = 0$

```
type Equation = float * float * float
type Solution = float * float
exception Solve;   (* declares an exception *)
```

```
let solve(a,b,c) =
      if b*b-4.0*a*c < 0.0 || a = 0.0 then raise Solve
      else ((-b + sqrt(b*b-4.0*a*c))/(2.0*a),
              (-b - sqrt(b*b-4.0*a*c))/(2.0*a));;
val solve : float * float * float -> float * float
```

The type of the function `solve` is (the expansion of)

`Equation -> Solution`

`d` is declared once and used 3 times    readability, efficiency

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# Solution using local declarations

```
let solve(a,b,c) =
    let d = b*b-4.0*a*c
    if d < 0.0 || a = 0.0 then raise Solve else
    ((-b + sqrt d)/(2.0*a),(-b - sqrt d)/(2.0*a));;
```

```
let solve(a,b,c) =
    let sqrtD =
      let d = b*b-4.0*a*c
      if d < 0.0 || a = 0.0 then raise Solve
      else sqrt d
    ((-b + sqrtD)/(2.0*a),(-b - sqrtD)/(2.0*a));;
```

Indentation matters

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```

Indentation matters

# Example: Rational Numbers

Consider the following signature, specifying operations and their types:

| **Specification** | **Comment** |
|---|---|
| type qnum = int * int | rational numbers |
| exception QDiv | division by zero |
| mkQ: int * int $\rightarrow$ qnum | construction of rational numbers |
| .+.: qnum * qnum $\rightarrow$ qnum | addition of rational numbers |
| .-.: qnum * qnum $\rightarrow$ qnum | subtraction of rational numbers |
| .*.: qnum * qnum $\rightarrow$ qnum | multiplication of rational numbers |
| ./.: qnum * qnum $\rightarrow$ qnum | division of rational numbers |
| .=.: qnum * qnum $\rightarrow$ bool | equality of rational numbers |
| toString: qnum $\rightarrow$ string | String representation of rational numbers |

# Intended use

```
let q1 = mkQ(2,3);;
let q2 = mkQ(12, -27);;
let q3 = mkQ(-1, 4) .*. q2 .-. q1;;
let q4 = q1 .-. q2 ./. q3;;
toString q4;;
val it : string = "-2/15"
```

$q_1 = \frac{2}{3}$

$q_2 = -\frac{12}{27} = -\frac{4}{9}$

$q_3 = -\frac{1}{4} \cdot q_2 - q_1 = -\frac{5}{9}$

$q_4 = q_1 - q_2 / q_3 = \frac{2}{3} - \frac{-4}{9} / \frac{-5}{9}$

$= -\frac{2}{15}$

Operators are infix with usual precedences

Note: Without using infix:

```
let q3 = (.-.)((.*.) (mkQ(-1,4)) q2) q1;;
```

# Intended use

let q1 = mkQ(2,3);; $q_1 = \frac{2}{3}$

let q2 = mkQ(12, -27);; $q_2 = -\frac{12}{27} = -\frac{4}{9}$

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let q4 = q1 .-. q2 ./. q3;; $q_4 = q_1 - q_2 / q_3 = \frac{2}{3} - \frac{-4}{9} / \frac{-5}{9}$

toString q4;;
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| toString q4;; | |
| *val it : string = "-2/15"* | $= -\frac{2}{15}$ |

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# Representation: $(a, b)$, $b > 0$ and $\texttt{gcd}(a, b) = 1$

Example $-\frac{12}{27}$ is represented by $(-4, 9)$

Greatest common divisor (Euclid's algorithm)

| | |
|---|---|
| `let rec gcd = function`<br>`  \| (0,n) -> n`<br>`  \| (m,n) -> gcd(n % m,m);;`<br>`val gcd : int * int -> int` | `- gcd(12,27);;`<br>`val it : int = 3` |

Function to cancel common divisors:

```
let canc(p,q) =
   let sign = if p*q < 0 then -1 else 1
   let ap = abs p
   let aq = abs q
   let d  = gcd(ap,aq)
   (sign * (ap / d), aq / d);;

canc(12,-27);;
val it : int * int = (-4, 9)
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# Program for rational numbers

Declaration of the constructor:

```
exception QDiv;;
let mkQ = function
   | (_,0)   -> raise QDiv
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Rules of arithmetic:

$$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd} \qquad \frac{a}{b} - \frac{c}{d} = \frac{ad-bc}{bd}$$

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd} \qquad \frac{a}{b} / \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} \quad \text{when } c \neq 0$$

$$\frac{a}{b} = \frac{c}{d} = ad = bc$$

Program corresponds direly to these rules

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let (.+.) (a,b) (c,d) = canc(a*d + b*c, b*d);;
let (.-.) (a,b) (c,d) = canc(a*d - b*c, b*d);;
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Note: Functions must preserve the invariant of the representation

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# Pattern matching and recursion

Consider `unzip` that maps a list of pairs to a pair of lists:

$$\begin{aligned}&\texttt{unzip}([(x_0, y_0); (x_1, y_1); \ldots; (x_{n-1}, y_{n-1})]\\ &\quad = ([x_0; x_1; \ldots; x_{n-1}], [y_0; y_1; \ldots; y_{n-1}])\end{aligned}$$

with the declaration:

```
let rec unzip = function
    | []            -> ([],[])
    | (x,y)::rest -> let (xs,ys) = unzip rest
                     (x::xs,y::ys);;

unzip [(1,"a");(2,"b")];;
val it : int list * string list = ([1; 2], ["a"; "b"])
```

Notice

- pattern matching on result of recursive call
- `unzip` is polymorphic. Type?
- `unzip` is available in the `List` library.

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$$\texttt{unzip([}(x_0, y_0)\texttt{;}(x_1, y_1)\texttt{;}\ldots\texttt{;}(x_{n-1}, y_{n-1})\texttt{]}$$
$$= \texttt{([}x_0\texttt{;}x_1\texttt{;}\ldots\texttt{;}x_{n-1}\texttt{],[}y_0\texttt{;}y_1\texttt{;}\ldots\texttt{;}y_{n-1}\texttt{])}$$

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# Summary

You are acquainted with a major part of the F# language.

- Higher-order (or curried) functions
- Basic types, equality and ordering
- Polymorphism
- Tuples
- Patterns
- A look at lists and list recursion